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## The effect of spin alignment on the metal–insulator transition in two-dimensional systems

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**Abstract.** We propose a mechanism for the observed suppression of the two-dimensional (2D) conducting phase when there is an in-plane magnetic field. We apply our approach, which is based on the memory function formalism, to the spin-polarized electron system. This takes into account both disorder and exchange–correlation effects. We show that spin polarization significantly favours localization because of the enhancement of the exchange–correlations. A key outcome is that the conducting phase for the fully spin-polarized system is suppressed. The in-plane magnetic field needed to generate the fully spin-polarized state is of the order of 1 T and depends on the carrier density. We determine the metal–insulator phase boundary for the unpolarized and polarized systems, and we estimate the dependence of the critical magnetic field on carrier density.

### 1. Introduction

Even though the existence of a metal–insulator transition for two-dimensional electron systems [1] has been known of for several years, the nature of the insulating and metallic states near the transition is still a puzzle. In the presence of a magnetic field which is perpendicular to the electron plane, the familiar quantum Hall states are recovered [2]. This is due to the dominant contribution of orbital effects in the magneto-conductance. If the magnetic field is parallel to the electron plane it can only couple directly to the electron spin. Recent experiments [3–5] have reported that a weak parallel magnetic field is sufficient to destroy the conducting phase, making the system insulating. The critical magnetic field needed varies with the carrier density but is of the order of 1 T for both Si and GaAs.

At the relatively high densities found in conventional metals and semiconductors the electron correlations resulting from mutual Coulomb repulsion are not important because the average interaction energies are much smaller than the Fermi energies. Thus the transport properties of conventional metals and semiconductors are well accounted for by the standard *nearly free-electron* picture. However, without electron repulsion the 2D system would always be insulating in the presence of disorder [6], so it is clear that to treat the metal–insulator transition we must look beyond this picture. At low electron densities the Fermi energy is small and the correlations start to dominate. Strong electron correlations drive the system to an insulating state (the Wigner crystal) even in the absence of any disorder when the electron density is sufficiently low [7]. The fact that the metal–insulator transition occurs at higher densities than the predicted Wigner transition has been interpreted as the additional effect of electron–defect interactions. Defects reduce the mobility of the electrons thus making them easier to localize. However, the nature of the insulating state at low densities in the presence

of low levels of disorder remains an unresolved issue and one which needs to be resolved if the metal–insulator transition is to be understood.

In contrast with the impurity-driven Anderson or independent-particle localization, Wigner-crystal-type localization is a phenomenon driven by the correlations between electrons. Due to the correlations, in each Wigner–Seitz cell there is one electron surrounded by a region of zero electron density as if the electron had a hard repulsive core. The complete density profile of the exchange–correlation hole is given by the two-electron correlation function  $g(r)$ . This function has been determined by diffusion quantum Monte Carlo numerical simulations (DQMC) of the pure electron system without imperfections in the substrate [7, 8]. The simulations have determined that the Wigner crystal melts at  $r_s \simeq 35$  and that when the electron density is increased the radius of the hard core in the exchange–correlation hole shrinks continuously.  $r_s$  is the dimensionless measure of density, equal to the average electron spacing in units of effective Bohr radii. There is a hard core for all densities  $r_s \gtrsim 7$ , a density 25 times higher than the density at the Wigner transition.

Wigner localization has close parallels with the close-packing solidification of charged hard spheres where the dominant mechanism for the solidification arises from fluctuations in the density variable. We have shown that when there are low levels of defects, electron localization at low electron densities is still driven by the electron correlations [9]. The impurities act as pinning centres and facilitate the localization. There is no long-range order when there is disorder and the electron solid is a frozen glass with liquid-like short-range order. We have found in the range of electron densities  $r_s \gtrsim 10$  that weak substrate disorder causes a glass transition.

The DQMC numerical simulations establish that the exchange–correlation hole is stronger for spin-polarized electrons than for unpolarized electrons. This is because there is additional exchange for the polarized system between the increased number of parallel-spin electrons. For at least  $r_s \lesssim 20$  the ground state continues to be the unpolarized electron liquid [8], but from  $r_s > 10$  the free energies for the spin-polarized state are very close to the free energies for the unpolarized state. Thus the Zeeman energy gain from a relatively weak parallel magnetic field should produce a fully polarized ground state. The polarized state is more likely than the unpolarized state to be in the insulating state because of the enhancement in the exchange–correlation hole. We are proposing this as the mechanism for the destabilization of the conducting phase by parallel magnetic fields.

We focus here on the localization of polarized and unpolarized strongly correlated systems in the presence of weak disorder. As we move away from the limit of very strong correlations where the density fluctuations  $\rho_q(t) = \sum_{k < k_F} a_{k+q}^\dagger(t) a_k(t)$  cause Wigner localization, then to leading order the basis set continues to be the density fluctuations. In our formalism, defect scattering is incorporated in such a way that the Ward identities and particle conservation are satisfied within this density basis. By restricting the basis to density fluctuations we average over phase information of processes where the particles and holes propagate independently, and thus exclude the possibility of Anderson localization which is not found in the limit of low densities.

## 2. Theory

We discuss the metal–insulator transition through the density correlation function  $\langle \rho_q(t) \rho_0(0) \rangle$  which provides the time dependence of the decay of the density fluctuations. In the liquid phase  $\langle \rho_q(t) \rho_0(0) \rangle$  goes to zero when  $t \rightarrow \infty$  because of the propagation of the diffusive mode. In the glass phase the diffusion constant is zero, so if we approach the glass transition the decay of density fluctuations becomes very slow and eventually stops. The local structure and local

density fluctuations no longer vanish when  $t \rightarrow \infty$ , and  $\langle \rho_q(t)\rho_0(0) \rangle$  does not go to zero. A narrow quasi-elastic peak builds up in the dynamic structure factor  $S(q, \omega)$ , so  $S(q, \omega)$  develops a singularity at zero frequency.

We have previously established that strong correlations in the presence of weak disorder in the substrate can localize the electrons into a glassy state [9]. We have obtained good agreement with the position of the metal–insulator transition in zero magnetic field for unpolarized electrons [10]. In our approach we search for the metal–glass transition using the Kubo-relaxation function

$$\Phi_\nu(q, t) \equiv \langle N_q(t) | N_0(0) \rangle. \quad (1)$$

This determines the relaxation of the normalized dynamical density fluctuations for polarization  $\nu$ ,  $N_q(t) = \rho_q(t) / \sqrt{\chi_\nu(q)}$ , as a function of time. When the index  $\nu = p$  the system is fully spin polarized while  $\nu = u$  is for the unpolarized system.  $\chi_\nu(q)$  is the corresponding static susceptibility.

We are interested in the dynamics of relaxation processes as  $t \rightarrow \infty$ . The order parameters for the glassy states are given by the limiting form of the relaxation function,  $f_\nu(q) = \lim_{t \rightarrow \infty} \Phi_\nu(q, t)$ . The Laplace transform of  $\Phi_\nu(q, t)$  is

$$\Phi_\nu(q, z) = \left( N_q \left| \frac{1}{\mathcal{L} - z} \right| N_q \right) \quad (2)$$

and the corresponding long-time limit of  $\Phi_\nu(q, t)$  is

$$f_\nu(q) = - \lim_{z \rightarrow 0} z \Phi_\nu(q, z). \quad (3)$$

When  $f_\nu(q)$  is non-zero, spontaneous fluctuations do not decay even at infinite time and the system becomes a frozen electron glass. In the conducting phase,  $f_\nu(q)$  is identically zero since the density fluctuations decay to zero.

The Liouvillian  $\mathcal{L}$  in equation (2) is for the Hamiltonian

$$H = \sum_k \epsilon_k a_k^\dagger a_k + \frac{1}{2} \sum_q V(q) \rho_q \rho_{-q} + \sum_q U(q) \rho_{-q}. \quad (4)$$

$\epsilon_k = \hbar^2 k^2 / 2m^*$  is the single-particle kinetic energy,  $\rho_q = \rho_q(0)$ ,  $V(q)$  is the Coulomb interaction between electrons, and  $U(q)$  is the defect potential.

We now introduce a subspace of the dynamical variables which spans the slowly varying dynamical variables of the system. In our case this consists of all the density fluctuations. Using the equation of motion for the density fluctuations, we can express  $\Phi_\nu(q, z)$  in terms of force–force relaxation functions  $M_\nu(q, z)$ . Using mode-coupling theory the  $M_\nu(q, z)$  can be approximated by linear and bi-linear products of the  $\Phi_\nu(q, z)$ . The result is a set of non-linear equations which self-consistently calculate the  $f_\nu(q)$ .

The projection operators

$$\begin{aligned} \mathcal{P} &= |N_q\rangle\langle N_q| & \bar{\mathcal{P}} &= 1 - \mathcal{P} \\ \mathcal{J} &= |\mathcal{L}N_q\rangle\langle \mathcal{L}N_q| \frac{1}{(\mathcal{L}N_q|\mathcal{L}N_q)} & \tilde{\mathcal{J}} &= 1 - \mathcal{J} \end{aligned} \quad (5)$$

project all dynamical variables onto the subspace spanned by density fluctuations  $|N_q\rangle$ , and current-density fluctuations  $|\mathcal{L}N_q\rangle$ , respectively.  $\mathcal{P}$  and  $\mathcal{J}$  satisfy the identities (see, for example, reference [11])

$$\mathcal{P} = \left[ \mathcal{P} \mathcal{L} \mathcal{P} - z - \mathcal{P} \mathcal{L} \bar{\mathcal{P}} \frac{1}{\bar{\mathcal{P}} \mathcal{L} \bar{\mathcal{P}} - z} \bar{\mathcal{P}} \mathcal{L} \mathcal{P} \right] \mathcal{P} \frac{1}{\mathcal{L} - z} \mathcal{P} \quad (6)$$

$$\mathcal{J} = \left[ \mathcal{J} \tilde{\mathcal{L}} \mathcal{J} - z - \mathcal{J} \tilde{\mathcal{L}} \tilde{\mathcal{J}} \frac{1}{\tilde{\mathcal{J}} \tilde{\mathcal{L}} \tilde{\mathcal{J}} - z} \tilde{\mathcal{J}} \tilde{\mathcal{L}} \mathcal{J} \right] \mathcal{J} \frac{1}{\tilde{\mathcal{L}} - z} \mathcal{J}. \quad (7)$$

Here  $\tilde{\mathcal{L}} \equiv \bar{\mathcal{P}}\mathcal{L}\bar{\mathcal{P}}$  is the reduced Liouvillian in the density subspace. Taking a scalar product of equation (6) with the density fluctuation  $|N_q\rangle$  we obtain

$$1 = \left[ (N_q|\mathcal{P}\mathcal{L}\mathcal{P}|N_q) - (N_q|z|N_q) - \left( N_q \left| \mathcal{P}\mathcal{L}\bar{\mathcal{P}} \left| \frac{1}{\bar{\mathcal{P}}\mathcal{L}\bar{\mathcal{P}} - z} \right| \bar{\mathcal{P}}\mathcal{L}\mathcal{P} \right| N_q \right) \right] \\ \times (N_q|\mathcal{P}|N_q) \left( N_q \left| \frac{1}{\mathcal{L} - z} \right| N_q \right) (N_q|N_q). \quad (8)$$

Using the definition of  $\Phi_v(q, z)$  we get

$$\Phi_v(q, z) = -\frac{1}{z + K_v(q, z)} \quad (9)$$

where the current relaxation function

$$K_v(q, z) = \left( \mathcal{L}N_q \left| \frac{1}{\tilde{\mathcal{L}} - z} \right| \mathcal{L}N_q \right) \quad (10)$$

describes the relaxation of the current fluctuation variable  $|\mathcal{L}N_q\rangle$ . For notational brevity we now drop the polarization index  $v$  except where it is explicitly necessary. In deriving equation (9) we have used  $(N_q|\mathcal{L}|N_q) = 0$  and  $\bar{\mathcal{P}}\mathcal{L}|N_q\rangle = |\mathcal{L}N_q\rangle$ . Both of these relations are the result of time-reversal symmetry.

Taking a scalar product of equation (7) with current fluctuation  $|\mathcal{L}N_q\rangle$  we get

$$(\mathcal{L}N_q|\mathcal{J}|\mathcal{L}N_q) = \left( N_q\mathcal{L} \left[ \mathcal{J}\tilde{\mathcal{L}}\mathcal{J} - z - \mathcal{J}\tilde{\mathcal{L}}\bar{\mathcal{J}} \frac{1}{\bar{\mathcal{J}}\tilde{\mathcal{L}}\bar{\mathcal{J}} - z} \bar{\mathcal{J}}\tilde{\mathcal{L}}\mathcal{J} \right] \right| \mathcal{L}N_q \right) \\ \times \frac{1}{(\mathcal{L}N_q|\mathcal{J}\mathcal{L}N_q)} \left( \mathcal{L}N_q \left| \frac{1}{\tilde{\mathcal{L}} - z} \right| \mathcal{L}N_q \right). \quad (11)$$

Solving for  $K(q, z)$  we obtain

$$K(q, z) = -\frac{\Omega_q}{z + M(q, z)} \quad (12)$$

where

$$\Omega_q = (\mathcal{L}N_q|\mathcal{L}N_q) = \chi^{-1}(q)(\mathcal{L}\rho_q|\mathcal{L}\rho_q) = \chi^{-1}(q)(q^2/m).$$

We have used  $\mathcal{J}|\mathcal{L}N_q\rangle = |\mathcal{L}N_q\rangle$ , and  $(\mathcal{L}N_q|\tilde{\mathcal{L}}|\mathcal{L}N_q) = 0$ , once again the result of time-reversal symmetry. The force-force relaxation function  $M(q, z)$  describes the relaxation of the reduced force variable  $|\bar{\mathcal{P}}\mathcal{L}^2\rho_q\rangle$ . Using  $\mathcal{P}|\mathcal{L}N_q\rangle = 0$  and  $|\bar{\mathcal{J}}\tilde{\mathcal{L}}\mathcal{L}N_q\rangle = |\bar{\mathcal{P}}\mathcal{L}^2N_q\rangle$ ,  $M(q, z)$  reduces to

$$M(q, z) = \frac{m}{q^2} \left( \bar{\mathcal{P}}\mathcal{L}^2\rho_q \left| \frac{1}{\bar{\mathcal{J}}\tilde{\mathcal{L}}\bar{\mathcal{J}} - z} \right| \bar{\mathcal{P}}\mathcal{L}^2\rho_q \right). \quad (13)$$

One needs to determine  $|\bar{\mathcal{P}}\mathcal{L}^2\rho_q\rangle$  for our Hamiltonian (equation (4)). Using the equation of motion for  $\rho_q$

$$\mathcal{L}^2\rho_q = \frac{1}{2m} \sum_k [q^2 + 2\mathbf{q} \cdot \mathbf{k}] a_{q+k}^\dagger a_k + \frac{1}{m} \sum_k V(k)(\mathbf{q} \cdot \mathbf{k}) \rho_k \rho_{q-k} + \frac{1}{m} \sum_k U(k)(\mathbf{q} \cdot \mathbf{k}) \rho_{q-k} \quad (14)$$

equation (13) becomes

$$M(q, z) = \frac{1}{mq^2} \left( \left[ \sum_k V(k)(\mathbf{q} \cdot \mathbf{k}) \rho_k \rho_{q-k} + \sum_k U(k)(\mathbf{q} \cdot \mathbf{k}) \rho_{q-k} \right] \bar{\mathcal{P}} \left| \frac{1}{\bar{\mathcal{J}}\tilde{\mathcal{L}}\bar{\mathcal{J}} - z} \right| \bar{\mathcal{P}} \right. \\ \left. \times \left[ \sum_k V(k)(\mathbf{q} \cdot \mathbf{k}) \rho_k \rho_{q-k} + \sum_k U(k)(\mathbf{q} \cdot \mathbf{k}) \rho_{q-k} \right] \right). \quad (15)$$

For weak scattering from defects we neglect the cross terms in equation (15), obtaining the expression

$$M(q, z) = M_{ee}(q, z) + M_{de}(q, z) \quad (16)$$

where  $M_{ee}(q, z)$  is the contribution from electron–electron scattering and  $M_{de}(q, z)$  is the contribution from electrons scattering from defects.  $M_{de}(q, z)$  is a two-point density relaxation function:

$$M_{de}(q, z) = \frac{1}{mq^2} \sum_{kk'} \left( U(k)(\mathbf{q} \cdot \mathbf{k}) \rho_{\mathbf{q}-\mathbf{k}} \bar{\mathcal{P}} \left| \frac{1}{\bar{\mathcal{J}} \bar{\mathcal{L}} \bar{\mathcal{J}} - z} \right| \bar{\mathcal{P}} \rho_{\mathbf{q}-\mathbf{k}'} U(k')(\mathbf{q} \cdot \mathbf{k}') \right) \quad (17)$$

which we approximate as

$$\begin{aligned} M_{de}(q, t) &= \frac{1}{mq^2} \sum_{kk'} \left\langle \left( U(k)(\mathbf{q} \cdot \mathbf{k}) U(k')(\mathbf{q} \cdot \mathbf{k}') \right) (\rho_{\mathbf{q}-\mathbf{k}}(t) | \rho_{\mathbf{q}-\mathbf{k}'}(0)) \right\rangle \\ &= \frac{n_i}{mq^2} \sum_{\mathbf{k}} U^2(k)(\mathbf{q} \cdot \mathbf{k})^2 \chi(|\mathbf{q} - \mathbf{k}|) \Phi(|\mathbf{q} - \mathbf{k}|, t). \end{aligned} \quad (18)$$

$M_{ee}(q, z)$  is a four-point relaxation function given by

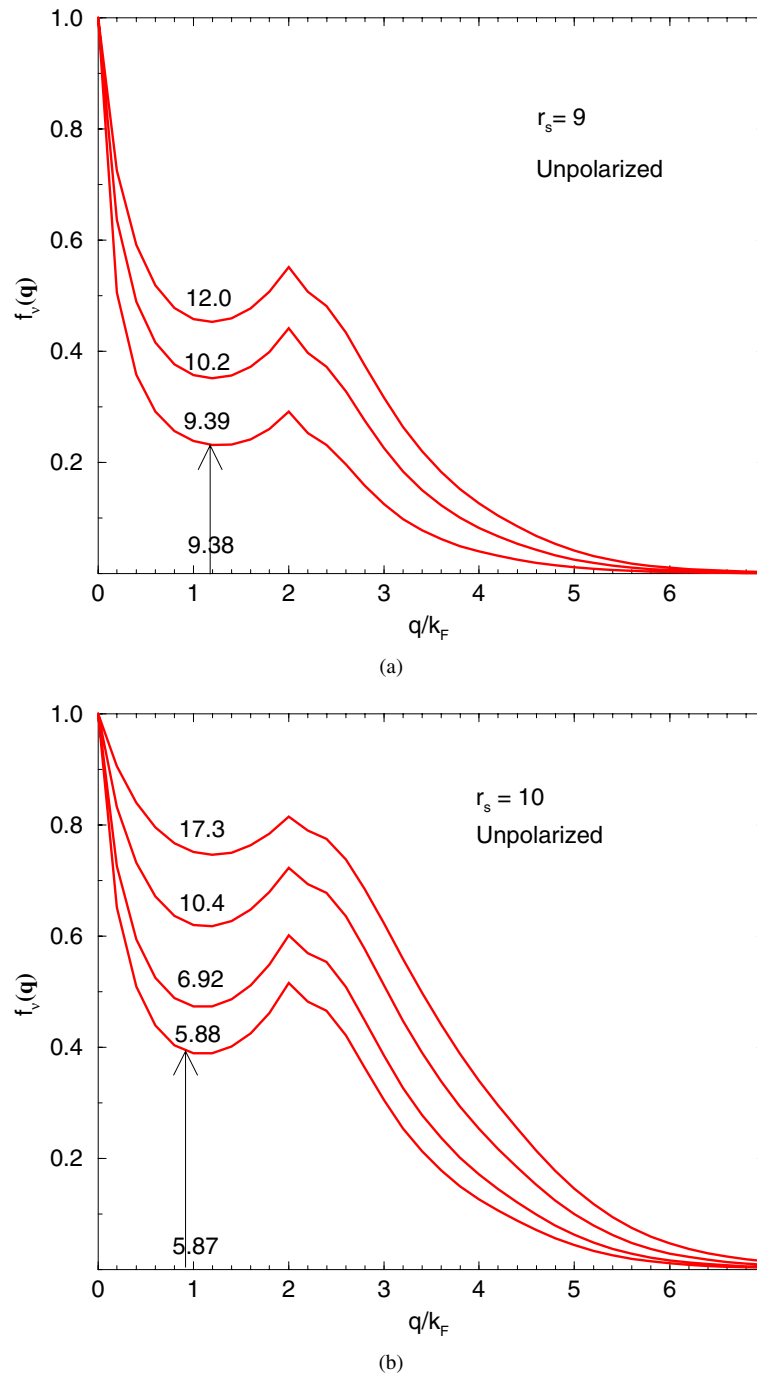
$$M_{ee}(q, z) = \frac{1}{mq^2} \sum_{kk'} \left( V(k)(\mathbf{q} \cdot \mathbf{k}) \rho_{\mathbf{k}} \rho_{\mathbf{q}-\mathbf{k}} \bar{\mathcal{P}} \left| \frac{1}{\bar{\mathcal{J}} \bar{\mathcal{L}} \bar{\mathcal{J}} - z} \right| \bar{\mathcal{P}} \rho_{\mathbf{k}'} \rho_{\mathbf{q}-\mathbf{k}'} V(k')(\mathbf{q} \cdot \mathbf{k}') \right). \quad (19)$$

One customarily proceeds from this point by factorizing four-point density correlation functions as products of two-point correlation functions. For a classical system this implies that the four-point relaxation function  $M_{ee}(q, z)$  in equation (19) can also be factorized, since the relaxation and correlation functions are equivalent, but this does not automatically follow in a quantum system since the two functions are not identical except in the high-temperature limit,  $\langle A|B \rangle_{\omega} = (1 - e^{-\beta \hbar \omega})^{-1} (A|B)_{\omega}$ . In order to identify the transition, however, we only need the limit  $\omega \rightarrow 0$ . Since in this limit the two functions are identical, we can factorize the quantum four-point relaxation function in terms of two-point relaxation functions. Equation (19) then becomes

$$\begin{aligned} M_{ee}(q, t) &= \frac{1}{mq^2} \sum_{kk'} \left\langle V(k)(\mathbf{q} \cdot \mathbf{k}) V(k')(\mathbf{q} \cdot \mathbf{k}') \right\rangle \\ &\quad \times \left[ \langle \rho_{\mathbf{k}}(t) \rho_{\mathbf{k}'}(0) \rangle \langle \rho_{\mathbf{q}-\mathbf{k}}(t) \rho_{\mathbf{q}-\mathbf{k}'}(0) \rangle + \langle \rho_{\mathbf{k}}(t) \rho_{\mathbf{q}-\mathbf{k}'}(0) \rangle \langle \rho_{\mathbf{q}-\mathbf{k}}(t) \rho_{\mathbf{k}'}(0) \rangle \right] \\ &= \frac{1}{mq^2} \sum_{kk'} \left\langle V(k)(\mathbf{q} \cdot \mathbf{k}) V(k')(\mathbf{q} \cdot \mathbf{k}') \right\rangle \\ &\quad \times \left[ \Phi(\mathbf{k}, \mathbf{k}', t) \chi(\mathbf{k}, \mathbf{k}') \Phi(\mathbf{q} - \mathbf{k}, \mathbf{q} - \mathbf{k}', t) \chi(\mathbf{q} - \mathbf{k}, \mathbf{q} - \mathbf{k}') \right. \\ &\quad \left. + \Phi(\mathbf{k}, \mathbf{q} - \mathbf{k}', t) \chi(\mathbf{k}, \mathbf{q} - \mathbf{k}') \Phi(\mathbf{q} - \mathbf{k}, \mathbf{k}', t) \chi(\mathbf{q} - \mathbf{k}, \mathbf{k}') \right]. \end{aligned} \quad (20)$$

For  $t \rightarrow \infty$  (or equivalently for  $\lim_{z \rightarrow 0}$ ), equations (16), (18), and (20) become

$$\begin{aligned} M(q) &\equiv M_{ee}(q) + M_{de}(q) \\ M_{ee}(q) &= \frac{1}{2mq^2} \sum_{q'} \left[ V(q')(\mathbf{q} \cdot \mathbf{q}') + V(|\mathbf{q} - \mathbf{q}'|)(\mathbf{q} \cdot (\mathbf{q} - \mathbf{q}')) \right]^2 \\ &\quad \times \chi(q') f(q') \chi(|\mathbf{q} - \mathbf{q}'|) f(|\mathbf{q} - \mathbf{q}'|) \\ M_{de}(q) &= \frac{n_i}{mq^2} \sum_{q'} U^2(q')(\mathbf{q} \cdot \mathbf{q}')^2 \chi(|\mathbf{q} - \mathbf{q}'|) f(|\mathbf{q} - \mathbf{q}'|). \end{aligned} \quad (21)$$



**Figure 1.** Order parameters  $f_v(q)$  for  $r_s = 9$  and 10. Curve labels are in-plane impurity densities  $n_i$  in units of  $10^9 \text{ cm}^{-2}$ . (a) Unpolarized;  $r_s = 9$ .  $f_u(q) = 0$  when  $n_i < 9.39 \times 10^9 \text{ cm}^{-2}$ . (b) Unpolarized;  $r_s = 10$ .  $f_u(q) = 0$  when  $n_i < 5.88 \times 10^9 \text{ cm}^{-2}$ . (c) Fully polarized;  $r_s = 9$ .  $f_p(q) = 0$  when  $n_i < 2.56 \times 10^9 \text{ cm}^{-2}$ . (d) Fully polarized;  $r_s = 10$ .  $f_p(q) = 0$  when  $n_i < 0.69 \times 10^9 \text{ cm}^{-2}$ .

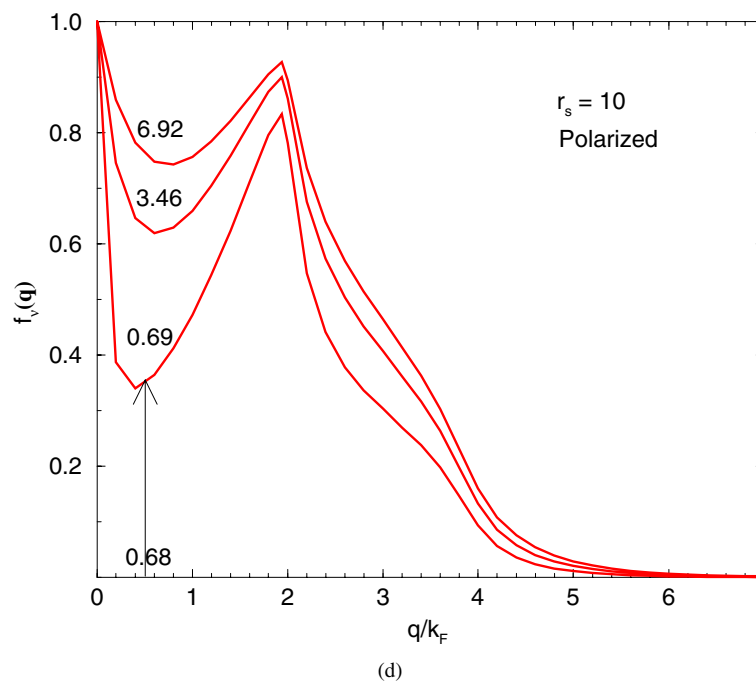
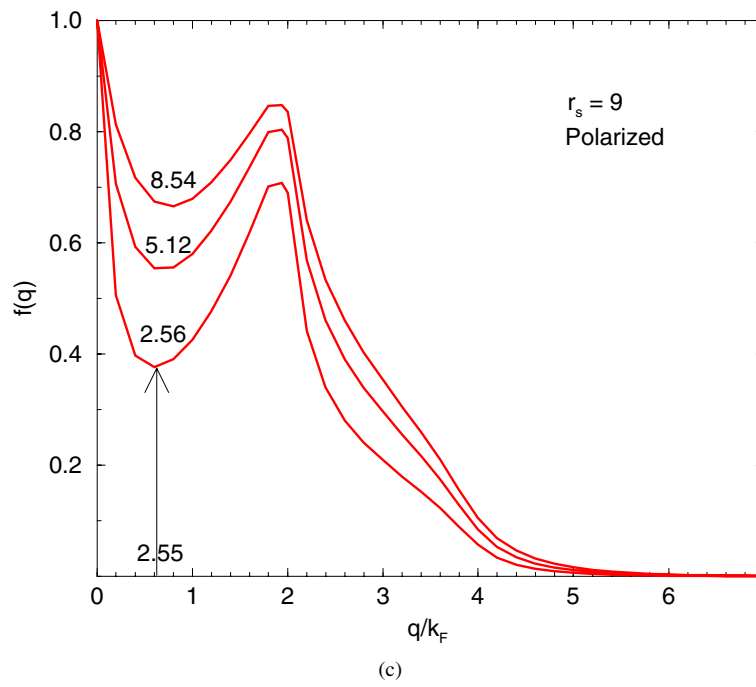


Figure 1. (Continued)

Using equations (3), (9), and (12), we write the order parameter  $f_v(q)$  in terms of  $M_v(q)$ :

$$f_v(q) = \frac{1}{1 + \Omega_q/M_v(q)}. \quad (22)$$



$M_v(q)$  itself depends on  $f_v(q)$ , and so combining equations (21) and (22) leads to a closed set of non-linear equations for  $f_v(q)$ . We solve these iteratively until the solutions are self-consistent.

The electron correlations are taken into account through the static susceptibilities  $\chi_v(q)$  in the vertex part of the memory function (equation (21)). The  $\chi_v(q)$  feed in the information about exchange and correlations. We have previously established that the key property in  $\chi_v(q)$  determining the transition is the size of the area occupied by the total density exclusion region in the centre of the exchange–correlation hole. We assume here when the electron correlations are strong that the shape of the exchange–correlation hole is not greatly affected when there are low levels of disorder. We take  $\chi_v(q)$  from the DQMC static structure factor  $S(q)$  [7, 8]. We introduce the static local field factor  $G_v(q)$  which we determine using

$$S_v(q) = \int_0^\infty d\omega \frac{\chi_v^{(0)}(q, i\omega)}{1 + V(q)[1 - G_v(q)]\chi_v^{(0)}(q, i\omega)}. \quad (23)$$

The  $G_v(q)$  then determine  $\chi_v(q)$  using (see reference [12])

$$\chi_v(q) = \chi_v^{(0)}(q)\{1 + V(q)[1 - G_v(q)]\chi_v^{(0)}(q)\}^{-1}. \quad (24)$$

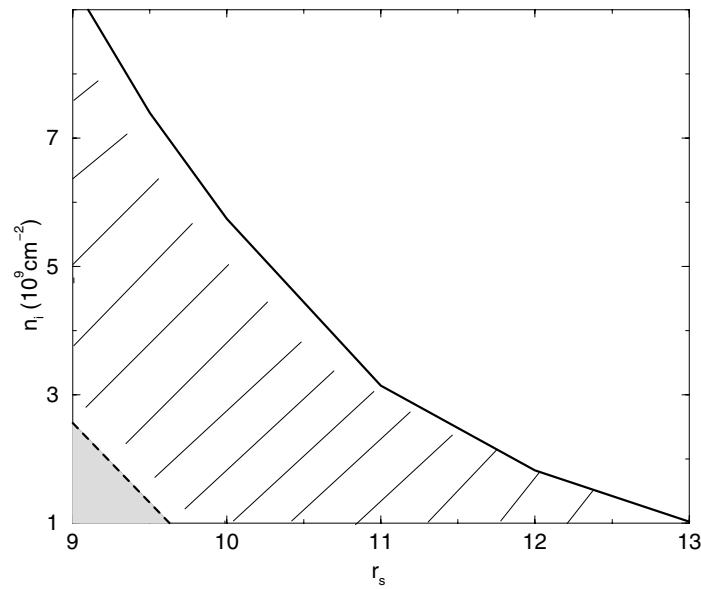
$\chi_v^{(0)}(q)$  is the Lindhard function for non-interacting electrons. The level of disorder is measured in terms of the scattering rate  $\gamma_v$  for carriers scattering from the defects. We evaluate  $\gamma_v$  [10] using the memory function formalism [13], and from this we determine the conductivity at the transition using the Drude relation.

### 3. Results

In figure 1 we show the order parameters  $f_v(q)$  determined from equations (21) and (22) for the polarized and unpolarized states for a range of impurity densities  $n_i$ . We consider scattering off monovalent Coulomb impurities randomly distributed in the carrier plane so that, in equation (21),  $U(q) = 2\pi e^2/(\epsilon q)$ . When  $n_i$  is less than a critical density,  $f_v(q)$  is zero, indicating a conducting phase. The present formalism provides no further information about its nature.  $f_v(q) = 0$  is equally consistent with a conducting phase which is a Fermi liquid or a conducting phase which is a superconductor (see reference [14]). At the critical  $n_i$ , we find the order parameter  $f_v(q)$  jumps discontinuously, indicating a transition to an insulator which is a frozen glass state [9]. The key point here is that for fixed  $r_s$  the critical impurity density is much smaller for the fully polarized system than it is for the unpolarized system.

We have determined the critical  $n_i$  as a function of the carrier density for both the polarized and unpolarized cases. Figure 2 shows the resulting phase boundaries between conductor and insulator. We have established that the conducting phase does not persist below  $r_s = 8$  [10]. The conducting phase for the fully polarized system which is represented by the shaded region is restricted to a small range of  $r_s$  below  $r_s \simeq 10$ . The conducting phase exists only for small levels of disorder. For the unpolarized system at the same  $r_s$  the critical level of disorder is significantly greater, and the conducting phase extends to much larger values of  $r_s$ . The hatched region represents the reduction in the conducting phase region when going from the unpolarized to the fully polarized system. Figure 2 thus shows that fully spin polarizing the system destabilizes the conducting phase except within a very small range of carrier densities. The stable conducting phase is restricted to very small levels of disorder. This significant shrinkage of the conducting region is associated with the enhancement of exchange–correlations for the fully polarized system. This enhancement favours localization.

We propose that the disappearance of the conducting phase in the presence of an in-plane magnetic field is associated with polarization of the carrier spins. At these low carrier densities



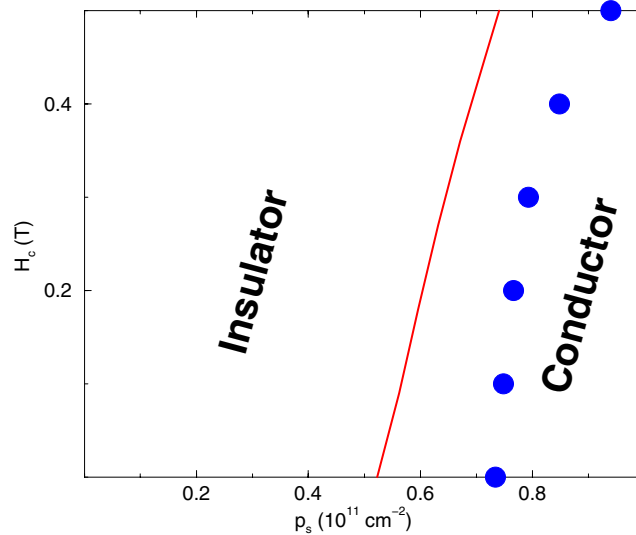
**Figure 2.** Phase boundaries between conducting and insulating states for unpolarized (solid line) and fully polarized systems (dashed line). Axes are impurity density  $n_i$  and electron  $r_s$ . For a fully polarized system the shaded region is conducting, and the remaining area is insulating. For an unpolarized system the conducting phase is the hatched region, which includes the shaded area, and the remainder is insulating.

the energy cost for spin-aligned states becomes small and a weak magnetic field is sufficient to fully polarize the electrons. DQMC simulations [8] have found that the ground-state energies  $E_p$  and  $E_u$  for the fully polarized and unpolarized systems respectively differ by only a small amount for densities below  $r_s \gtrsim 10$ . From the  $E_p$  and  $E_u$  we can calculate the critical magnetic field  $H_c$  needed to change the unpolarized ground state into a fully spin polarized state. We equate the Zeeman energy splitting at the critical field  $H_c$  with the energy difference  $H_c = (E_p - E_u)\hbar/(g\mu_B)$ . We use  $(g\sigma_z) = 1.1$  for holes in GaAs, taken from reference [15].

Hamilton *et al* [5] reported for a GaAs sample with hole density  $p_s$  corresponding to  $r_s = 9$  that a magnetic field  $\lesssim 0.7$  T drives the conducting state to an insulator. We find that at  $r_s = 9$  the critical magnetic field needed to fully polarize the system is  $H_c = 0.6$  T, which is very close to this value. For electrons in Si MOSFETs the values of effective mass and  $(g\sigma_z)$  are not very different from those for holes in GaAs. The value of  $H_c = 0.5$  T in Si measured by Simonian *et al* [3] at  $r_s = 9$  is again in good agreement with our value.

We find at  $r_s = 9$  that the critical disorder level needed to drive the fully polarized system to the insulating state corresponds to a conductivity of  $\sigma \simeq 4.5e^2/h$ . This is consistent with the measured value at the transition of  $\sigma \simeq 5e^2/h$  for  $r_s = 9$  [5]. Figure 2 shows the effect of spin polarization on the phase boundary for an impurity concentration corresponding to  $\sigma = 5e^2/h$ . The value of  $r_s$  at the phase boundary decreases when the system goes from the unpolarized to the fully spin-polarized state. It drops from  $r_s = 11.5$  to  $r_s = 9.2$ .

Reference [5] determines the metal–insulator phase boundary as a function of hole density  $p_s$  and critical magnetic field  $H_c$ . To compare with the experimental points we use a linear interpolation between  $p_s$  and  $H_c$  to determine the critical magnetic field as a function of hole density (solid line). Figure 3 compares the experimental points with our calculated  $H_c$ . We find reasonable agreement.



**Figure 3.** The dependence of the critical magnetic field for the metal–insulator transition in GaAs on the hole density  $p_s$  (solid line) for impurity density corresponding to  $\sigma = 5e^2/h$ . Points are experimental data taken from Hamilton *et al* [5].

In conclusion, we have demonstrated that magnetic fields  $\sim 1$  T should be sufficient to fully spin polarize the carriers for  $r_s > 8$  and we have shown that the enhanced exchange–correlations for the fully polarized system significantly favour the insulating phase. Our mechanism leads to results which are in reasonable quantitative agreement with experiment. Figure 2 predicts a re-emergence of a conducting phase for the fully polarized system at very small levels of disorder for a narrow range of carrier densities.

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